GENERATION OF TIME DOMAIN DRIVE SIGNALS IN MIMO RANDOM VIBRATION CONTROL SYSTEM

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ABSTRACT

The new generation algorithm of time domain drive signals is presented to reduce the loop time in MIMO (multi input multi output) random vibration control system. The Parks-McClellan method is used to design the FIR (finite impulse response) filter. And then the drive signals in time domain is generated by filtering a series of independent white noise with the designed filter. For avoiding the time domain randomization process in the conventional frequency-to-time transformation, the new method is favorable to improve the real-time property of the control system. Two tests are run in one 6 DOF (degree of freedom) hydraulic shaker with the conventional algorithm and the improved algorithm differently. The results demonstrate the effectiveness of the improved algorithm.

KEY WORDS

Shaker, MIMO random vibration, FIR filter

NOMENCLATURE

\( f \): frequency
\( B(f) \): the error PSD between the reference and the control signals
\( G_n(f) \): the averaged PSD of control signals
\( G_m(f) \): the averaged PSD of drive signals
\( G_{nm}(f) \): the averaged cross-spectral density of the control and drive signals
\( G_n(f)_j \): the drive signals PSD after \( j \) times iterations
\( G_n(f)_{j+1} \): the drive signals PSD after \( j+1 \) times iterations
\( H(f) \): frequency response function matrix
\( Z(f) \): impedance matrix
\( \alpha \): iteration step size
\( \psi(f) \): phase frequency characteristic of the FIR filter
\( \theta(f) \): phase frequency characteristic of the drive signals PSD
\( \varphi(f) \): linear phase
INTRODUCTION

The shaker is always used for the simulation of actual vibrations to adequately test devices prior to their actual use. The vibrations caused by many natural phenomena and man-made systems are random in nature and are not restricted to specific frequencies. These vibrations are often defined in terms of statistical quantities such as the acceleration spectral density or PSD\(^1\). In order to adequately test many devices prior to their actual use, it is necessary that the PSD, the devices be subjected in actual usage, are replicated accurately in the shaker. Due to the limit of the frequency bandwidth and the nonlinearity of the system, the servo control can’t meet the needs of the PSD replication precision. So the vibration control algorithm is used to improve the control precision of PSD replication\(^2\).

FISHER & POSEHN\(^3\) preformed some early work on MIMO random control in 1977. The interactive closed-loop control algorithm of random vibration was presented in their paper. Smallwood and his co-workers\(^4\) significantly contributed to the progress in MISO (multi-input single-output) random control. Stroud & HAMMA\(^2\) generalized the progress of vibration control in 1988 and presented the swept-sine and random vibration algorithms with single shaker and multi shakers. Underwood\(^5-6\) presented the adaptive control method of MIMO swept-sine in 1994.

In the conventional PSD replication algorithm, the drive signals in time domain is generated by frequency domain randomization and time domain randomization. The time domain randomization consists of delay, reversal, windowing and overlapping of the pseudo-random signal generated by frequency domain randomization. For the complexity of time domain randomization, the conventional PSD replication algorithm takes a long time to generate the drive signals in time domain.

The new generation algorithm of time domain drive signals is presented in this paper to reduce the loop time in random vibration test. With the information contained in the drive signals PSD, the Parks-McClellan method is used to design the FIR filter. The drive signals in time domain is generated by filtering a series of independent white noise with the designed filter. Figure 2 shows the principle of the filter method.

Figure 1 Generation of drive signals with time domain randomization

It can be seen from Figure 1 that the conventional method consists of 4 steps.

1. Cholesky decomposition: Convert the drive PSD to amplitude spectrum with choleseky decompose.
2. Frequency domain randomization: Generate the random phase of Gauss distribution and convert the amplitude spectrum to spectrum.
3. Inverse Fourier transformation: Convert the spectrum to pseudo-random signals with inverse Fourier transformation.
4. Time domain randomization: Convert the pseudo-random signals to true-random signals through delay, reversal, windowing and overlapping.

For the complexity of time domain randomization, the classical PSD replication algorithm takes a long time to generate the drive signals in time domain.

Filter Method

With the information contained in the drive signals PSD, the Parks-McClellan method is used to design the FIR filter. The drive signals in time domain is generated by filtering a series of independent white noise with the designed filter. Figure 2 shows the principle of the filter method.

Figure 2 Generation of drive signals with FIR filter

From the filter design theory we know that filters designed using the Parks-McClellan method have equal ripple in their passbands and stopbands. For this reason,
they are often called equiripple filters. They represent the most efficient filter designs for a given specification, meeting the frequency response specification with the lowest order filter[8]. The filters are optimal in the sense that they minimize the maximum error between the desired frequency response and the actual frequency response.

From the stochastic process theory we know that

\[ G_{i\alpha}(f) = H(f)G_{i\alpha}(f)H(f)^H \]  

(1)

If the drive signal is white noise, the value of \( G_{i\alpha}(f) \) is constant. Assuming \( G_{i\alpha}(f) = a \), we can get

\[ |H(f)| = \sqrt{G_{i\alpha}(f)/a} \]  

(2)

\( |H(f)| \) is just the amplitude frequency response characteristics of the FIR filter. If the PSD of input white noise signal is \( a \), the PSD \( G_{i\alpha}(f) \) of the output signals can be got when the FRF of the system satisfy Eq (2).

Adding the linear phase \( \phi \) to the phase frequency characteristics of the drive signals PSD, we can get the phase frequency characteristics of the designed FIR filter as

\[ \psi(f) = \theta(f) + \phi(f) \]  

(3)

\( \phi(f) \) is given by

\[ \phi(f) = -\frac{Mf}{2} \]  

(4)

Where \( M \) is the order of the FIR filter.

With the known amplitude and phase frequency response characteristics of the FIR filter, the \textit{remez} function in the signal processing toolbox in Matlab can be used to design the FIR filter using the Parks-McClellan method[9].

THE IMPROVED ALGORITHM OF MIMO RANDOM VIBRATION CONTROL

The schematic diagram of the improved random vibration algorithm is shown in Figure 3. The drive signals PSD is corrected by the impedance of the system and the deviation between the reference PSD and the control signals PSD, so that the reference PSD is replicated in high precision in the output of the system.

Figure 3 shows that the random vibration algorithm consists of the FRF estimation, the impedance computation, the correction of drive signals PSD and the generation of the drive signals in time domain.

FRF Estimation and Impedance Computation

Let the drive signals of \( n \) in \( n \) out system be \( \{d_i(t), i = 1, \ldots, n\} \) and the response signals be \( \{c_i(t), i = 1, \ldots, n\} \). The FRF H1 estimator is always used in vibration control system[11]. The H1 estimator is given by

\[ H(f) = G_{i\alpha}(f)G_{i0}(f)^{-1} \]  

(5)

\( H(f) \) is a \( n \times n \) complex matrix for every frequency. The inverse of FRF is called the impedance. Let \( Z(f) \) be the impedance, we can get

\[ Z(f) = [H(f)]^{-1} \]  

(6)

Correction of Drive Signals PSD

Defining the reference PSD as \( R(f) \) and the control signals PSD after \( j \) times iteration as \( G_{i\alpha}(f)_j \), we can get the error PSD as

\[ E(f)_j = R(f) - G_{i\alpha}(f)_j \]  

(7)

The drive signals PSD is corrected by

\[ G_{i\alpha}(f)_{j+1} = G_{i\alpha}(f)_j + \alpha Z(f)E(f)_j[Z(f)]^H \]  

(8)

The value of \( \alpha \) needs to be corrected by operator during the tests. Generally speaking, if the estimated FRF matrix matches the true FRF within some acceptable error margin, Eq.(8) is convergent with the \( \alpha \) value chosen between 0 and 1. The global convergence property of Eq.(8) is waiting to be proved[5].

Generation of Time Domain Drive Signals
Based on Eq.(2), the remez function is used to design the FIR filter with the information contained in the drive signals PSD. And then the drive signals in time domain is generated by filtering a series of independent white noise with the designed filter.

**Accuracy Test**

The two indexes used to test the precision of random vibration are the RMS value and the frequency domain error between the control and reference PSD. The former is used to test the energy difference of the two PSD. The latter is used to compute the frequency domain errors of the two PSD and is more visualized and specific than the former. So we take the latter index to test the precision of random vibration. The frequency domain error \( B(f) \) is given by

\[
B(f) = \frac{G_m(f)}{R(f)}
\]  

(9)

The nearer \( B(f) \) is to 1, the higher the random vibration precision is. Chinese standard\(^{[12]}\) specify that \( B(f) \) should be controlled within \( \pm 3 \text{dB} \) in single axis random vibration test.

**TEST AND RESULTS**

Two MIMO random vibration tests are run in one 6 DOF hydraulic shaker. The first test is to generate time domain drive signals with time domain randomization. And the second test is to generate time domain drive signals with FIR filter.

The structure of the shaker is shown in Figure 4. The three translation direction is defined as \( x, y \) and \( z \). The three rotation direction is defined as \( Rx \), \( Ry \) and \( Rz \).

The reference PSD is generated by MATLAB and the shape is the same in 6 DOF. The reference PSD consists of three segments. The rising spectrum is from 2 to 5Hz. The flat spectrum is from 5 to 60Hz. The ascent spectrum is from 60 to 70Hz. The PSD of flat spectrum is \( 3 \times 10^{-4} \text{g}^2/\text{Hz} \) in translation and \( 2.1 \times 10^{-3} \text{(rad/s)}^2/\text{Hz} \) in rotation.

Figure 5 shows the amplitude frequency response characteristic of the diagonal elements of the measured FRF matrix. Where \( H_{11} \) presents the amplitude frequency response characteristic of the element located in first row first column of the FRF matrix. It means the amplitude frequency response characteristic of \( x \) DOF.

For the amplitude frequency response characteristic of FRF matrix has different deviation to 0 dB in different frequency, we know that the control would has great bias to the reference if with no compensation. So the MIMO random vibration control algorithm is needed to improve the control precision. For the sake of simplify, the test results in \( z \) and \( Rz \) direction are only shown in Figures 6-7.

Figure 6 shows the results after twice iterations in the first test in which the drive signal is generated with time domain randomization. Figure 7 shows the results after twice iterations in the second test in which the time domain drive signals is generated by filtering a series of independent white noise with the filter designed by Parks-McClellan method. It can be seen from Figures 6-7 that the control PSD has been controlled within \( \pm 3 \text{dB} \) tolerance of the reference PSD in the whole frequency band. Excellent test result has been achieved in these two tests. But the time spent in the generation of time domain drive signals in second test is less half than that spent in first test. So the loop time is reduced greatly in second test.
From the comparison between Figure 6 and Figure 7 we know that the control PSD in the first test is smoother than that in second test. The drive signals generated by time domain randomization have some pseudo-randomness. The energy of the control signals generated by these drive signals converges in the original frequency certainly. While the drive signals generated by filter is the random signals in true means. The energy of the drive signals is evenly distributed in the whole frequency band of the pass band of the designed filter. This is the main reason of the poor flatness. But the control precision is high enough to satisfy the requirement of MIMO random vibration.

CONCLUSIONS

A new algorithm is presented to generate time domain drive signals. The Parks-McClellan method is used to design the FIR filter and the drive signals in time domain is generated by filtering a series of independent white noise with the designed filter. Test results verify the validity of the new method.

The performance of the improved MIMO random vibration control algorithm is verified by a realistic simulation. The excellent test results indicate that the improved algorithm is favorable to reduce the loop time in MIMO random vibration control system.

REFERENCES

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